The pressure difference $(p_s - p)$ just beneath the cutting surface can be found by applying Darcy's law for the flow of fluid through a porous medium:

$$\underset{\sim}{v} - \underset{\sim}{u} = k \, \nabla p \, . \tag{19}$$

Darcy's law has been written in a special form to accommodate the motion of the rock in the present coordinates. k is the permeability of the rock, $\underset{\sim}{v}$ is the vector velocity, and $\underset{\sim}{u}$ is the volume flux of fluid through the pores. $\underset{\sim}{u}$ can be understood by imagining a small pillbox cut into the rock broadside to the flow. $\underset{\sim}{u}$ would then be the velocity of the fluid through the pillbox.

Now consider the saturated region CSDW in Fig. 6, and assume that the sides CW and SD have some depth δ small compared with the other dimensions like d (the proof that δ is small will be given shortly). Since WD is an air-water interface, no flow can take place across it in the coordinates chosen. Whatever flow takes place through the narrow sides CW and SD is negligible, so flow into the cutting surface CS must be essentially zero to conserve water volume. The same argument applies to any surface parallel to CS, and the component of $\underset{\sim}{u}$ normal to the cutting surface is therefore zero throughout the saturated region CSDW. If the rock feeds under the jet at a speed v, then the normal component of rock velocity is $v \sin \theta$ as seen in Fig. 6. The normal component of Darcy's law (19) is therefore

$$v \sin \theta = k \frac{\partial p}{\partial n} \, .$$

The pore pressure beneath the cutting surface follows by integration:

$$p_s - p = \begin{cases} \dfrac{|n| \; v \sin \theta}{k} \, , & |n| < \delta \; ; \\[2ex] p_s - p_a \, , & |n| > \delta \; . \end{cases} \tag{20}$$

The absolute value $|n|$ is used to avoid minus signs, since the normal coordinate n is negative on the rock side of the cutting surface.

The criterion for failure of the cutting surface is based on a hypothesis fundamental to this theory: <u>the rock is in a continuous state of incipient fracture one grain diameter g beneath the cutting surface</u>. Setting $|n| = g$ and combining (18) and (20), one arrives at the following expression for the shear stress on the rock side of the cutting surface:

$$\tau = \tau_o + \mu_r \frac{gv}{k} \sin \theta \, . \tag{21}$$